TWO-BODY OSCILLATION

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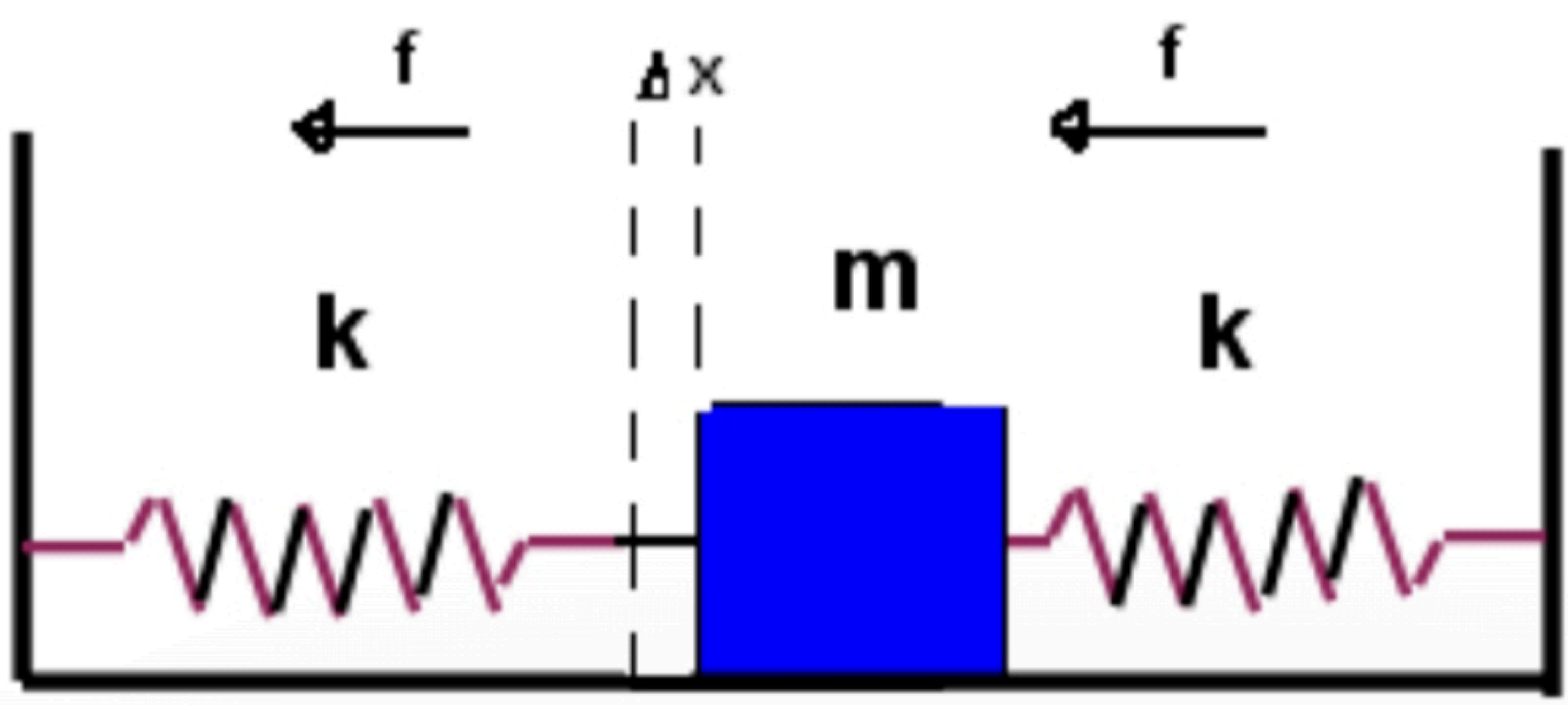
1. Definition

Two-body oscillation refers to the oscillatory motion of a system consisting of **two masses connected by an elastic restoring force**, such as a spring, where **both bodies are free to move**. The motion of each mass influences the other, and the system oscillates about its equilibrium position.

2. Physical Description of the System

Consider two particles of masses m_1 and m_2, connected by a **light, ideal spring** of force constant k.

- The system is placed on a smooth horizontal surface.
- There is **no external force** acting on the system.
- Oscillations occur due to internal restoring forces only.



3. Assumptions

1. The spring is massless and obeys **Hooke's law.**
2. Motion is one-dimensional.
3. No damping or friction is present.
4. Oscillations are small, hence linear.

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4. Coordinates and Displacements

Let:

- x_1 = displacement of mass m_1
- x_2 = displacement of mass m_2

The extension of the spring is:

$$x = x_2 - x$$

5. Equations of Motion

For mass m_1:

$$m_1\ddot{x}_1 = k(x_2 - x_1)$$

For mass m_2:

$$m_2\ddot{x}_2 = -k(x_2 - x_1)$$

Subtracting the two equations gives the equation for relative motion:

$$\ddot{x} + \frac{k(m_1+m_2)}{m_1m_2}x = 0$$

6. Reduced Mass Concept

Define **reduced mass** μ as:

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Then the equation becomes:

$$\ddot{x} + \frac{k}{\mu} x = 0$$

This is the standard equation of **simple harmonic motion (SHM)**.

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7. Angular Frequency of Two-Body Oscillation

$$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$$

8. Time Period of Oscillation

$$T = 2\pi\sqrt{\frac{\mu}{k}} = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1+m_2}}$$

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9. Motion of the Centre of Mass

Since no external force acts on the system:

- The **centre of mass moves with constant velocity**.
- Oscillation occurs **relative to the centre of mass**.

If the system is initially at rest, the centre of mass remains stationary.

10. Special Cases

(i) One Mass Very Large ($m_2 \gg m_1$)

$$\mu \approx m_1$$

The system behaves like:

- A single mass m_1
- Oscillating against a fixed wall

This reduces to **ordinary SHM**.

(ii) Equal Masses ($m_1 = m_2 = m$)

$$\mu = \frac{m}{2}$$

$$\omega = \sqrt{\frac{2k}{m}}$$

Both masses oscillate with equal amplitude in opposite directions.

11. Energy of the System

Kinetic Energy:

$$KE = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$$

Potential Energy:

$$PE = \frac{1}{2}k(x_2 - x_1)^2$$

Total energy remains **constant**.

12. Physical Significance

- Two-body oscillation introduces the idea of **relative motion**.
- It explains vibrations in **molecules**, **crystal lattices**, and **diatomic systems**.
- The concept of reduced mass is crucial in **quantum mechanics** and **molecular physics**.

13. Conclusion

Two-body oscillation is an important extension of simple harmonic motion where **both bodies are movable**. By transforming the system into relative coordinates, the motion reduces to SHM with reduced mass. This model plays a fundamental role in understanding **molecular vibrations, coupled oscillators, and advanced physical systems**.